

# Algebra, Number theory and Combinatorics

## Overall Contest

**Question 1.** Let  $G$  be a non-abelian finite  $p$ -group ( $p$  being a prime). Show that  $[G : G'] \geq p^2$ , where  $G' = [G, G]$ .

**Question 2.** Suppose that  $B$  is a non-degenerate symmetric bilinear form on  $V = \mathbf{C}^n$ ,  $T$  is a nilpotent linear operator on  $V$  whose Jordan form is a single block, such that  $B(Tv, w) = -B(v, Tw)$  for all  $v, w \in V$ . Show that  $n$  is odd.